Uniform Asymptotic Approximation and Inflationary Perturbation Spectra in the effective field theory of inflation

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Motivations: EFT of inflation?

Observational supports a single scalar field inflation with a self-interacting potential, however

- sensitive to high energy physics
- precise observational data may probe the new physics corrections

$$\epsilon_*^n \sim \left(\frac{H}{M_*}\right)^n \tag{1}$$

The effective field theory of inflation can provides

- a general framework for describing the most generic single scalar field theory
- a natural approach to explore the high energy physics and their observational predictions

The action of the EFT of inflation: [JHEP 03 (2008) 014]

$$S_{\rm eft} = M_{\rm Pl}^2 \int d^4x\sqrt{-g}\left\{\frac{R}{2} + \dot{H}g^{00} - (3H^2+\dot{H}) + \frac{M_2^4}{2M_{\rm Pl}^2}(g^{00}+1)^2 \right.$$
$$\left. + \frac{\bar{M}_1^3}{2M_{\rm Pl}^2}(g^{00}+1)\delta K_\mu^\mu - \frac{\bar{M}_2^2}{2M_{\rm Pl}^2}(\delta K_\mu^\mu)^2 - \frac{\bar{M}_3^2}{2M_{\rm Pl}^2}\delta K_\nu^\mu \delta K_\mu^\nu \right\},$$

The equations for scalar mode u_k and tensor mode v_k

$$u_k'' + \left(\omega_k^2(\eta) - \frac{z''}{z}\right)u_k = 0, \tag{2}$$

$$v_k'' + \left(c_t^2(\eta)k^2 - \frac{a''}{a}\right)v_k = 0 \tag{3}$$

where for scalar mode,

$$\omega_k^2(\eta) = c_s^2 k^2\left(1 - \frac{A_2}{A_0 c_s^2}\frac{k^2}{a^2}\right). \tag{4}$$

Extended EFT of inflation: adding high derivative terms to S_{eff}

$$\Delta S = \int d^4x\sqrt{-g}\Big\{ \frac{\bar{M}_4}{2}\nabla g^{00}\nabla^\nu \delta K_{\mu\nu} - \frac{\delta_1}{2}(\nabla_\mu \delta K^{\nu\gamma})(\nabla^\mu \delta K_{\nu\gamma})$$
$$- \frac{\delta_2}{2}(\nabla_\mu \delta K^\nu_\nu)^2 - \frac{\delta_3}{2}(\nabla_\mu \delta K^\mu_\nu)(\nabla_\gamma \delta K^{\gamma\nu}) - \frac{\delta_4}{2}\nabla^\mu \delta K_{\nu\mu}\nabla^\nu \delta K^\sigma_\sigma \Big\}.$$

The equations for scalar mode u_k becomes

$$u_k'' + \left(\omega_k^2(\eta) - \frac{z''}{z}\right)u_k = 0, \qquad (5)$$

where the dispersion relation changes into

$$\omega_k^2(\eta) = c_s^2 k^2 \left(1 - \frac{D_1}{G_1 c_s^2}\frac{k^2}{a^2} + \frac{C_1}{G_1 c_s^2}\frac{k^4}{a^4}\right). \qquad (6)$$

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A long-standing problem

It is in general impossible to get the exact inflationary observables analytically in inflation models.

$$\mu_k''(\eta) + \left(\omega_k^2(\eta) - \frac{z''}{z}\right)\mu_k(\eta) = 0$$

Some alternative approximations:

- Bessel function approximation [Schwarz et. al, PLB517(2001)243];
- Green's function method [Stewart and Gong, PLB510(2001)1;Wei, Cai, and Wang, PLB603(2004)95];
- WKB approximation [Martin and Schwarz, PRD67,083512(2005); Casadio et.al., PLB625(2005)1];
- Improved WKB approximation [Casadio et. al. PRD72,103516(2005)];
- Phase integral method [Rojas and Villalba, PRD 75, 063518 (2007)];
- Uniform asymptotic approximation [Hibab et.al. PRL89, 281301(2002); PRD70,083507(2004); TZ, AZ, QW, et.al., PRD89,043507(2014); IJMPA29,1450142(2014); PRD90,063503(2014);PRD90,103517(2014); APJL 807(2015)L17; JCAP10(2015)052; JCAP03(2016)046; PRD93(2016)123525.]

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The uniform asymptotic approximation

- Treatments for one single turning point [Langer (1931, 1932, 1935, . . . 1949); Olver, Philos. Trans. R. Soc. A 247, 307 (1954); Asymptotics and Special Functions, (AKP Classics, Wellesley, MA 1997)]
  - Applications to inflationary cosmology [Habib et. al., PRL89,281301(2002); PRD70,083507(2004); PRD71,043518(2005); Martin, Ringeval, and Vennin, JCAP06 (2013) 021; Lorentz, Martin, and Ringeval, PRD78,083513(2008);TZ, AW, Cleaver, Kirsten, Sheng, QW, BFL, PRD90 (2014)063503; PRD90(2014)103517; APJL807(2015)L17; JCAP10(2015)052; JCAP03(2016)046; PRD97 (2018)103502; arXiv: 1812.11191]
- Treatments for two turning points problem [Olver, Phil. Trans. R. Soc. A 278, 137 (1975).]
  - Applications to inflationary cosmology [TZ, AW, Cleaver, Kirsten, Sheng, and QW, PRD89(2014)043507; IJMPA29(2014)1450142; PRD93(2016)123525, JCAP02 (2018)018; arXiv:1811.03216; arXiv: 1811.12612]
- Treatments with extreme points [TZ, AW, arXiv:1902.09675 [quant-ph]].

General formulas of power spectra for single turning point

$$\Delta^2(k) = \frac{k^2}{4\pi^2}\frac{-k\eta}{z^2(\eta)\nu(\eta)c_s(\eta)}\exp\left(2\int_y^{\bar{y}_0}\sqrt{g(\hat{y})}d\hat{y}\right) \times\left[1+\frac{\mathscr{H}(+\infty)}{\lambda}+\frac{\mathscr{H}^2(+\infty)}{2\lambda^2}+\dots\right]$$

where the error control function $\mathscr{H}(\xi)$ reads as

$$\frac{\mathscr{H}(\xi)}{\lambda} = \int_y^{y_0}\left(\frac{5}{16\hat{\xi}^3(y')}+\frac{q(y')}{\hat{g}(y')}-\frac{5\hat{g}'^2(y')}{16\hat{g}^3(y')}+\frac{\hat{g}''(y')}{4\hat{g}^2(y')}\right)\sqrt{\hat{g}(y')}dy'$$

Table: Errors to be expected in the uniform approximation

Quantity	1st-order	2nd-order	3th-order
Power spectrum: $\Delta^2(k)$	$\lesssim 15\%$	$\lesssim 1.5\%$	$\lesssim 0.15\%$

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General formulas of power spectra with extra two turning points

$$\Delta^2(k) \simeq \mathscr{A}(k)\frac{k^2}{4\pi^2}\frac{-k\eta}{z^2(\eta)\nu(\eta)}\exp\left(2\lambda\int_y^{y_0}\sqrt{\hat{g}(y')}dy'\right) \times\left[1+\frac{\mathscr{H}(+\infty)}{2\lambda}+\frac{\mathscr{H}^2(+\infty)}{8\lambda^2}+\mathcal{O}\left(\frac{1}{\lambda^3}\right)\right], \quad (7)$$

where $\mathscr{A}(k)$ denotes the modified factor due to the presence of the two extra turning points y_1 and y_2, which reads

$$\mathscr{A}(k) = 1+2e^{\pi\lambda\zeta_0^2}+2e^{\pi\lambda\zeta_0^2/2}\sqrt{1+e^{\pi\lambda\zeta_0^2}} \times\left\{\cos 2\mathfrak{B}-\frac{\mathscr{I}(\zeta)+\mathscr{H}(\xi)}{\lambda}\sin 2\mathfrak{B} -\frac{\left[\mathscr{I}(\zeta)+\mathscr{H}(\xi)\right]^2}{2\lambda^2}\cos 2\mathfrak{B}\right\}. \quad (8)$$

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Parameters in the EFT of inflation

- The EFT of inflation introduces a lot of new parameters $(M_2,\ \bar{M}_1,\ \bar{M}_2,\ \bar{M}_3)$, which are assumed to be small and slow-varying

$$\left|\frac{M_2^4}{M_{\rm Pl}^2 H^2 \epsilon_1}\right|, \left|\frac{\bar{M}_1^3}{M_{\rm Pl}^2 H \epsilon_1}\right|, \left|\frac{\bar{M}_2^2}{M_{\rm Pl}^2 \epsilon_1}\right|, \left|\frac{\bar{M}_3^2}{M_{\rm Pl}^2 \epsilon_1}\right| \ll 1.$$

- this leads to four sets of new slow-roll parameters

$$\delta_1 \equiv \frac{M_2^4}{M_{\rm Pl}^2 H^2 \epsilon_1},\ \ \sigma_1 \equiv \frac{\bar{M}_1^3}{M_{\rm Pl}^2 H \epsilon_1},\ \ \zeta_1 \equiv \frac{\bar{M}_2^2}{M_{\rm Pl}^2 \epsilon_1},\ \ \kappa_1 \equiv \frac{\bar{M}_3^2}{M_{\rm Pl}^2 \epsilon_1},,$$

$$\delta_{n+1} = \frac{d \ln \delta_n}{d \ln a},\ \ \sigma_{n+1} = \frac{d \ln \sigma_n}{d \ln a},\ \ \zeta_{n+1} = \frac{d \ln \zeta_n}{d \ln a},\ \ \kappa_{n+1} = \frac{d \ln \kappa_n}{d \ln a}.$$

Spectra at the second-order slow-roll approximation

The scalar and tensor perturbation spectra read

$$
\begin{aligned}
\Delta^2_{\mathcal{R}}(k) = \bar{A}_s\Big[&1+\bar{\delta}_1-\left(2-2\bar{D}_p\right)\bar{\epsilon}_1-\bar{D}_p\bar{\epsilon}_2-\frac{909\bar{\zeta}_1}{1448}-\frac{909\bar{\kappa}_1}{1448}-\frac{3\bar{\sigma}_1}{4}+\frac{3789\bar{\delta}_1\bar{\zeta}_1}{1448}-\frac{555\bar{\delta}_1\bar{\kappa}_1}{1448}+\frac{9\bar{\delta}_1\bar{\sigma}_1}{4}-\frac{\bar{\delta}_1^2}{2}\\
&+\left(-2\bar{D}_p-2\right)\bar{\delta}_1\bar{\epsilon}_1-\bar{D}_p\bar{\delta}_1\bar{\epsilon}_2+\left(\bar{D}_p+2\right)\bar{\delta}_2\bar{\delta}_1+\left(2\bar{D}_p^2+2\bar{D}_p+\bar{\Delta}_1+\frac{\pi^2}{2}-5\right)\bar{\epsilon}_1^2+\left(\frac{\bar{D}_p^2}{2}+\frac{\bar{\Delta}_1}{4}+\frac{\pi^2}{8}-\frac{3}{2}\right)\bar{\epsilon}_2^2\\
&+\left(\bar{D}_p^2-D_p+\bar{\Delta}_1+2\bar{\Delta}_2+\frac{7\pi^2}{12}-8\right)\bar{\epsilon}_1\bar{\epsilon}_2+\left(-\frac{\bar{D}_p^2}{2}+\bar{\Delta}_2+\frac{\pi^2}{24}\right)\bar{\epsilon}_2\bar{\epsilon}_3+\left(-\frac{909\bar{D}_p}{1448}-\frac{2007297}{524176}\right)\bar{\zeta}_1\bar{\zeta}_2\\
&+\left(-\frac{909\bar{D}_p}{1448}-\frac{958945}{524176}\right)\bar{\kappa}_1\bar{\kappa}_2+\left(-\frac{3\bar{D}_p}{4}-\frac{5}{2}\right)\bar{\sigma}_1\bar{\sigma}_2+\left(\frac{909\bar{D}_p}{724}+\frac{78105}{131044}\right)\bar{\zeta}_1\bar{\epsilon}_1+\left(\frac{909\bar{D}_p}{1448}-\frac{250953}{262088}\right)\bar{\zeta}_1\bar{\epsilon}_2\\
&+\left(\frac{909\bar{D}_p}{724}+\frac{78105}{131044}\right)\bar{\kappa}_1\bar{\epsilon}_1+\left(\frac{909\bar{D}_p}{1448}-\frac{250953}{262088}\right)\bar{\kappa}_1\bar{\epsilon}_2+\left(\frac{3D_p}{2}+\frac{3}{2}\right)\bar{\sigma}_1\bar{\epsilon}_1+\frac{3}{4}\bar{D}_p\bar{\sigma}_1\bar{\epsilon}_2-\frac{10323\bar{\zeta}_1\bar{\kappa}_1}{57920}\\
&-\frac{22581\bar{\zeta}_1\bar{\sigma}_1}{5792}-\frac{343683\bar{\zeta}_1^2}{115840}-\frac{861\bar{\kappa}_1\bar{\sigma}_1}{5792}+\frac{149277\bar{\kappa}_1^2}{115840}-\frac{57\bar{\sigma}_1^2}{32}\Big].
\end{aligned}
$$

$$
\begin{aligned}
\Delta^2_h(k) = \bar{A}_t\Big[&1+\left(-2\bar{D}_p-2\right)\bar{\epsilon}_1\\
&+\left(2\bar{D}_p^2+2\bar{D}_p+\bar{\Delta}_1+\frac{\pi^2}{2}-5\right)\bar{\epsilon}_1^2\\
&+\left(-\bar{D}_p^2-2\bar{D}_p+2\bar{\Delta}_2+\frac{\pi^2}{12}-2\right)\bar{\epsilon}_1\bar{\epsilon}_2-\frac{\bar{\epsilon}_1\bar{\kappa}_1}{2}\Big],
\end{aligned}
$$

[G.-H. Ding, Q. Jin, QW, TZ, AW, in preparation (2019)]

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Non-adiabatic evolutions

- The high derivative terms in the extended EFT of inflation leads to nonlinear dispersion relation

$$\omega_k^2(\eta) = c_s^2 k^2 \left(1 - \frac{D_1}{G_1 c_s^2}\frac{k^2}{a^2} + \frac{C_1}{G_1 c_s^2}\frac{k^4}{a^4}\right), \tag{11}$$

- which leads to a period of non-adiabatic evolution of scalar modes.
- This indicates that there are two extra turning points in the equation of motion.
- In the uniform asymptotic approximation, the non-adiabatic effects on the primordial spectra is measured by the modified factor $\mathscr{A}$.

Non-adiabatic effect on the spectra

- The power spectra is written in the form

$$\mathcal{P}_s = \mathscr{A} \times \mathcal{P}_s^{\mathrm{GR}}, \quad (12)$$

$$\mathscr{A} \simeq 1 + 2e^{\pi\zeta_0^2} + 2e^{\pi\zeta_0^2/2}\sqrt{1+e^{\pi\zeta_0^2}}\cos 2\mathfrak{B} \quad (13)$$

  - Strong violation of adiabaticity:
    $e^{\pi\zeta_0^2} \gg 1 \;\rightarrow\; \mathcal{A} \simeq 2e^{\pi\zeta_0^2}(1+\cos 2\mathfrak{B})$
  - weak violation of adiabaticity:
    $e^{\pi\zeta_0^2} \ll 1 \;\rightarrow\; \mathscr{A} \simeq 1 + 2e^{\pi\zeta_0^2}(1+\cos 2\mathfrak{B}) + \mathcal{O}(e^{\pi\zeta_0^2})$

- the scalar spectrum is still nearly scale invariant,

$$n_s \simeq n_s^{\mathrm{GR}} + \mathcal{O}\left(\frac{H^2}{M_*^2}\right). \quad (14)$$

[arXiv:1811.03216]

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Concluding remarks

- The uniform asymptotic approximation provides a precise and effective approach to describe the evolution of the primordial perturbation spectrum
- Using this method, we calculate for the first time the primordial perturbation spectra of the EFT of inflation up to the second-order slow-roll approximation;
- We also study in details the non-adiabatic evolution of primordial perturbation in the extended EFT of inflation and calculate the scalar spectrum;
- We find out that the spectrum with non-adiabatic effects is still nearly scale invariant.

Thanks!